

DECOMPOSITIONS OF MOTIVES
OF GENERALIZED SEVERI-BRAUER VARIETIES

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ABSTRACT. Let p be a positive prime number and X be a Severi-Brauer variety of a central division algebra D of degree p^n , with $n \geq 1$. We describe all shifts of the motive of X in the complete motivic decomposition of a variety Y , which splits over the function field of X and satisfies the nilpotence principle. In particular, we prove the motivic decomposability of generalized Severi-Brauer varieties $X(p^m, D)$ of right ideals in D of reduced dimension p^m , $m = 0, 1, \dots, n - 1$, except the cases $p = 2$, $m = 1$ and $m = 0$ (for any prime p), where motivic indecomposability was proven by Nikita Karpenko.

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CONTENTS

1. Introduction	152
2. Chow motives with finite coefficients	153
3. Main results	154
4. Complete motivic decompositions	159
References	164

1. INTRODUCTION

Let F be an arbitrary field and p be a prime number. For any integer l , we write $v_p(l)$ for the exponent of the highest power of p dividing l .

Let D be a central division F -algebra of degree p^n , with $n \geq 1$. We write $X(p^m, D)$ for the generalized Severi-Brauer variety of right ideals in D of reduced dimension p^m for $m = 0, 1, \dots, n$. In particular, $X(p^n, D) = \text{Spec } F$ and $X(1, D)$ is the usual Severi-Brauer variety of D . The generalized Severi-Brauer varieties are twisted forms of grassmannians (see [11, §I.1.C]).

For each integer $m = 0, \dots, n$ we define an *upper motive* $M_{m,D}$ in the category of Chow motives with coefficients in \mathbb{F}_p . This is the summand of the complete motivic decomposition of the variety $X(p^m, D)$ such that the 0-codimensional Chow group of $M_{m,D}$ is non-zero.

Let A be a central simple F -algebra, such that the p -primary component of A is Brauer equivalent to D . Let \mathfrak{X}_A be the class of finite direct products of projective $(\text{Aut } A)$ -homogeneous F -varieties (the class \mathfrak{X}_A includes the generalized Severi-Brauer varieties of the algebra A). Nikita Karpenko proved the following theorem [9, Theorem 3.8]. Any variety X from \mathfrak{X}_A decomposes into a sum of shifts of the motives $M_{m,D}$ with $m \leq v_p(\text{ind } A_{F(X)})$. This theorem shows that the motivic indecomposable summands $M_{m,D}$ of the generalized Severi-Brauer varieties $X(p^m, D)$ are some kind of “basic material” to construct the motives of more general class of varieties. This gives us a motivation to understand the structure of the upper motives $M_{m,D}$ themselves. It was known that in the cases $m = 0$ (Severi-Brauer case, see Corollary 3.2) and $m = 1$, $p = 2$ ([9, Theorem 4.2]) the motive $M_{m,D}$ coincides with the whole motive of the variety $X(p^m, D)$ (that is, the motive of this variety is indecomposable). Taking into account these cases and the fact that any generalized Severi-Brauer variety $X(p^m, D)$ is p -incompressible [9, Theorem 4.3] (this condition is weaker than motivic indecomposability), one probably expected that the Chow motive with coefficients in \mathbb{F}_p of any variety $X(p^m, D)$ is indecomposable. But, except the two already mentioned cases, the motivic decomposability of generalized Severi-Brauer variety $X(p^m, D)$ was proven in [14].

This article is an extended version of [14]. To show that the motive of the variety $X(p^m, D)$ is decomposable, we prove in [14] that some shifts of $M_{0,D}$ are the motivic summands of $X(p^m, D)$. Let Y be a F -variety satisfying the nilpotence principle and such that it splits over the function field of $X(1, D)$. For example, one can take for Y any generalized Severi-Brauer variety $X(p^m, D)$ and, more generally, any variety from \mathfrak{X}_A . The main result of the present article (Theorem 3.4) find all shifts of $M_{0,D}$ in the complete motivic decomposition of the variety Y in terms of some subgroups of rational cycles. These subgroups can be described in the case of generalized Severi-Brauer variety $X(p^m, D)$ (see Proposition 3.7). As consequence, we prove the motivic decomposability of these varieties in Corollary 3.8. With Theorem 3.4 in hand, we find in §4 more examples (comparing to [14]) of complete motivic decompositions of generalized Severi-Brauer varieties $X(p^m, D)$ and therefore we describe the upper

motives $M_{m,D}$ in that cases. Theorem 3.4 also permits to prove differently (see Corollary 3.12, [3, Corollary 5]) a particular case of the following conjecture.

CONJECTURE 1.1. *Let D be a central division F -algebra. Let K/F be a field extension such that D_K is still division. Then $(M_{m,D})_K$ is still indecomposable.*

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2. CHOW MOTIVES WITH FINITE COEFFICIENTS

A variety is a separated scheme of finite type over a field. Our basic reference for Chow groups and Chow motives (including notations) is [4]. We fix an associative unital commutative ring Λ . Given a variety X over a field F , we write $\text{Ch}(X)$ and $\text{CH}(X)$ respectively for its Chow group with coefficients in Λ and for its integral Chow group. For a field extension L/F we denote by X_L the respective extension of scalars. An element of $\text{Ch}(X_L)$ is called F -rational, if it lies in the image of the homomorphism $\text{Ch}(X) \rightarrow \text{Ch}(X_L)$.

Our category of motives is the category $\text{CM}(F, \Lambda)$ of graded Chow motives with coefficients in Λ , [4, definition of § 64]. By a sum of motives we always mean the direct sum. We also write Λ for the motive $M(\text{Spec}F) \in \text{CM}(F, \Lambda)$. A Tate motive is the motive of the form $\Lambda(i)$ with i an integer.

Let X be a smooth complete variety over F and let M be a motive. We call M split if it is a finite sum of Tate motives. We call X split, if its integral motive $M(X) \in \text{CM}(F, \mathbb{Z})$ (and therefore the motive of X with an arbitrary coefficient ring Λ) is split. We call M or X geometrically split, if it splits over a field extension of F . For a geometrically split variety X over F , we denote by \bar{X} the scalar extension of X to a splitting field of its motive and we write $\bar{\text{Ch}}(X)$ for the subring of F -rational cycles in $\text{Ch}(\bar{X})$. Note that the rings $\text{Ch}(\bar{X})$ and $\bar{\text{Ch}}(X)$ are independent on the choice of a splitting field.

Over an extension of F the geometrically split motive M becomes isomorphic to a finite sum of Tate motives. We write $\text{rk } M$ and $\text{rk}_i M$ for respectively the number of all summands and the number of summands $\Lambda(i)$ in this decomposition, where i is an integer. Note that these two numbers do not depend on the choice of a splitting field extension.

We say that X satisfies the nilpotence principle, if for any field extension E/F and any coefficient ring Λ , the kernel of the change of field homomorphism $\text{End}(M(X)) \rightarrow \text{End}(M(X)_E)$ consists of nilpotents. Any projective homogeneous (under an action of a semisimple affine algebraic group) variety is geometrically split and satisfies the nilpotence principle, [4, Theorem 92.4 with Remark 92.3].

A complete decomposition of an object in an additive category is a finite direct sum decomposition with indecomposable summands. We say that the Krull-Schmidt principle holds for a given object of a given additive category, if every direct sum decomposition of the object can be refined to a complete one (in particular, a complete decomposition exists) and there is only one (up to a permutation of the summands) complete decomposition of the object. We have the following theorem:

THEOREM 2.1. ([2, Theorem 3.6 of Chapter I]). *Assume that the coefficient ring Λ is finite. The Krull-Schmidt principle holds for any shift of any summand of the motive of any geometrically split F -variety satisfying the nilpotence principle.*

We will use the following two statements in the next section.

LEMMA 2.2. *Assume that the coefficient ring Λ is a field. Let X be a split variety. Then the bilinear form $\mathfrak{b} : \text{Ch}(X) \times \text{Ch}(X) \rightarrow \Lambda$, $\mathfrak{b}(x, y) = \deg(x \cdot y)$ is non-degenerate.*

Proof. Since the motive of X decomposes into a finite sum of Tate motives, we have the following decomposition for the diagonal class $\Delta \in \text{Ch}_{\dim X}(X \times X)$:

$$\Delta = a_1 \times b_1 + \dots + a_n \times b_n,$$

where a_1, \dots, a_n and b_1, \dots, b_n are the homogeneous elements in $\text{Ch}(X)$, such that for any $i, j = 1, \dots, n$ the degree $\deg(a_i \cdot b_j) \in \Lambda$ is 0 for $i \neq j$ and 1 for $i = j$.

Note that $\dim_{\Lambda} \text{Ch}(X) = \text{rk } M(X) = n < \infty$. Therefore, to prove the lemma it suffices to show that $\text{rad } \mathfrak{b} = \{0\}$. Suppose that $x \in \text{rad } \mathfrak{b}$ (this means $\mathfrak{b}(x, y) = 0$ for any $y \in \text{Ch}(X)$). Then we have

$$x = \Delta_*(x) = \sum_{i=1}^n \deg(x \cdot a_i) b_i = \sum_{i=1}^n \mathfrak{b}(x, a_i) b_i = 0.$$

□

LEMMA 2.3. *Assume that the coefficient ring Λ is finite. Let X be a variety satisfying the nilpotence principle. Let $f \in \text{End}(M(X))$ and $1_E = f_E \in \text{End}(M(X)_E)$ for some field extension E/F . Then $f^n = 1$ for some positive integer n .*

Proof. Since X satisfies the nilpotence principle, we have $f = 1 + \varepsilon$, where ε is nilpotent. Let n be a positive integer such that $\varepsilon^n = 0 = n\varepsilon$. Then $f^{n^n} = (1 + \varepsilon)^{n^n} = 1$ because the binomial coefficients $\binom{n^n}{i}$ for $i < n$ are divisible by n . □

3. MAIN RESULTS

Let p be a positive prime integer. The coefficient ring Λ is \mathbb{F}_p in this section. Let F be a field. Let D be a central division F -algebra of degree p^n . We

write $X(p^m, D)$ for the generalized Severi-Brauer variety of right ideals in D of reduced dimension p^m for $m = 0, 1, \dots, n$.

LEMMA 3.1. *Let E/F be a splitting field extension for $X = X(1, D)$. Then the subgroup of F -rational cycles in $\text{Ch}_{\dim X}(X_E \times X_E)$ is generated by the diagonal class.*

Proof. By [7, Proposition 2.1.1], we have $\bar{\text{Ch}}^i(X) = 0$ for $i > 0$. Since the (say, first) projection $X^2 \rightarrow X$ is a projective bundle, we have a (natural with respect to the base field change) isomorphism $\text{Ch}_{\dim X}(X^2) \simeq \text{Ch}(X)$. Passing to $\bar{\text{Ch}}$, we get an isomorphism $\bar{\text{Ch}}_{\dim X}(X^2) \simeq \bar{\text{Ch}}(X) = \bar{\text{Ch}}^0(X)$ showing that $\dim_{\mathbb{F}_p} \bar{\text{Ch}}_{\dim X}(X^2) = 1$. Since the diagonal class in $\bar{\text{Ch}}_{\dim X}(X^2)$ is non-zero, it generates all the group. \square

COROLLARY 3.2. (cf. [7, Theorem 2.2.1]). *The motive with coefficients in \mathbb{F}_p of the Severi-Brauer variety $X = X(1, D)$ is indecomposable.*

Proof. To prove that our motive is indecomposable it is enough to show that $\text{End}(M(X)) = \text{Ch}_{\dim X}(X \times X)$ does not contain nontrivial projectors. Let $\pi \in \text{Ch}_{\dim X}(X \times X)$ be a projector. By Lemma 3.1, π_E is zero or equal to 1_E . Since X satisfies the nilpotence principle, π is nilpotent in the first case, but also idempotent, therefore π is zero. Lemma 2.3 gives us $\pi = 1$ in the second case. \square

Nikita Karpenko proved the motivic indecomposability of generalized Severi-Brauer varieties also in the case $p = 2, m = 1$.

THEOREM 3.3. (cf. [9, Theorem 4.2]). *Let D be a central division F -algebra of degree 2^n with $n \geq 1$. Then the motive with coefficients in \mathbb{F}_2 of the variety $X(2, D)$ is indecomposable.*

Corollary 3.8 of the following main theorem will show that Corollary 3.2 and Theorem 3.3 give us the only cases when the motive of generalized Severi-Brauer variety is indecomposable.

THEOREM 3.4. *Let D be a central division F -algebra of degree p^n with $n \geq 1$. Let X be the Severi-Brauer variety $X(1, D)$ and Y be a variety satisfying nilpotence principle, such that Y is split over the function field of X . Then for any integer k the number of copies $M(X)(k)$ in the complete motivic decomposition of Y is equal to $\dim_{\mathbb{F}_p} f_* \text{Ch}_{\dim Y - k}(X \times Y)$, where f is a projection onto the second factor.*

Proof. We fix an integer k and we note the motive $M(X)(k)$ simply by M . Let r be the number of copies of M in the complete motivic decomposition of Y . We note $V := f_* \bar{\text{Ch}}_{\dim Y - k}(X \times Y)$ and $r' := \dim_{\mathbb{F}_p} V$. We want to show that $r = r'$.

Let A_1, \dots, A_m and B_1, \dots, B_n be the motives. We recall that a morphism between the motives $\bigoplus_{i=1}^m A_i$ and $\bigoplus_{j=1}^n B_j$ is given by an $n \times m$ -matrix of morphisms $A_i \rightarrow B_j$. The composition of morphisms is the matrix multiplication.

The motive $M^{\oplus r}$ is a summand of the motive $M(Y)$. Therefore there exist two morphisms $\alpha = (\alpha_1, \dots, \alpha_r)^t \in \text{Hom}(M^{\oplus r}, M(Y))$ and $\beta = (\beta_1, \dots, \beta_r) \in \text{Hom}(M(Y), M^{\oplus r})$, such that

$$\beta \circ \alpha = (\beta_j \circ \alpha_i)_{1 \leq i, j \leq r} = (\delta_{i,j})_{1 \leq i, j \leq r},$$

where $(\delta_{i,j})_{1 \leq i, j \leq r}$ is the identity morphism in $\text{Hom}(M^{\oplus r}, M^{\oplus r})$ (that is $\delta_{i,j}$ is zero if $i \neq j$ and $\delta_{i,i}$ is the diagonal class Δ in $\text{Corr}_0(X, X)$ if $i = j$).

Let $E = F(X)$, then E/F is a splitting field extension for the varieties X and Y (here we use the condition of the theorem) and $X_E \simeq \mathbb{P}^d$, where $d = p^n - 1$. We know that $\Delta_E = \sum_{i=0}^d h^i \times h^{d-i}$, where h is the hyperplane class in $\text{Ch}^1(X_E)$. For any $1 \leq i \leq r$ we have

$$\begin{aligned} (\beta_i)_E \circ (\alpha_i)_E &= (\delta_{i,i})_E = \Delta_E = \\ &= h^0 \times h^d + \sum_{i=1}^d h^i \times h^{d-i} = [X_E] \times [pt] + \sum_{i=1}^d h^i \times h^{d-i}, \end{aligned}$$

where $[pt]$ is the class of a rational point in $\text{Ch}(X_E)$. Therefore the correspondences $\beta_i \in \text{Ch}_{\dim Y - k}(Y_E \times X_E)$ and $\alpha_i \in \text{Ch}_{d+k}(X_E \times Y_E)$ have to be of the following form:

$$(3.5) \quad (\beta_i)_E = b_i \times [pt] + \dots,$$

where $b_i \in \text{Ch}^k(Y_E)$ is non-zero and where “...” stands for a linear combination of only those terms whose first factor has codimension $> k$,

$$(3.6) \quad (\alpha_i)_E = [X_E] \times b_i^* + \dots,$$

where $b_i^* \in \text{Ch}_k(Y_E)$ is such that $\deg(b_i \cdot b_i^*) = 1$ and where “...” stands for a linear combination of only those terms whose second factor has dimension $> k$. For any $i \neq j$ we have $(\beta_j)_E \circ (\alpha_i)_E = 0$, this implies that $\deg(b_j \cdot b_i^*) = 0$. Therefore the system of vectors $\{b_1^*, \dots, b_r^*\}$ from the vector space $\text{Ch}(Y_E)$ is dual to the system of vectors $\{b_1, \dots, b_r\}$ with respect to the bilinear form $\mathfrak{b} : \text{Ch}(Y_E) \times \text{Ch}(Y_E) \rightarrow \mathbb{F}_p$, $\mathfrak{b}(x_1, x_2) = \deg(x_1 \cdot x_2)$. It follows that the vectors b_1, \dots, b_r are linearly independent. Since $b_i = f_*((\beta_i^t)_E)$, then $b_i \in V$ for any $1 \leq i \leq r$. Therefore $r \leq r'$.

Let now $b_1, \dots, b_{r'}$ be a basis of V . We want to show that $M^{\oplus r'}$ is a motivic summand of Y . By the definition of V , there exist correspondences $\beta_1, \dots, \beta_{r'} \in \text{Ch}_{\dim Y - k}(Y \times X)$ of the form (3.5), such that $b_i = f_*((\beta_i^t)_E)$. Since the variety Y_E is split, then by Lemma 2.2 the bilinear form \mathfrak{b} is non-degenerate. It follows that there exists a system of vectors $\{b_1^*, \dots, b_{r'}^*\}$ from the vector space $\text{Ch}(Y_E)$, which is dual to the system of vectors $\{b_1, \dots, b_{r'}\}$. For any $1 \leq i \leq r'$ we construct the correspondence $\alpha_i \in \text{Ch}_{d+k}(X \times Y)$, such that $(\alpha_i)_E$ is of the form (3.6), by the following way. The pull-back homomorphism

$$g : \text{Ch}(X \times Y) \rightarrow \text{Ch}(Y_{F(X)}) = \text{Ch}(Y_E)$$

with respect to the morphism $Y_{F(X)} = (\text{Spec } F(X)) \times Y \rightarrow X \times Y$ given by the generic point of X is surjective by [4, Corollary 57.11]. We define

$\alpha_i \in \text{Ch}(X \times Y)$ as a cycle whose image in $\text{Ch}(Y_E)$ under the surjection g is b_i^* . We have $(\alpha_i)_E = [X_E] \times b_i^* + \dots$, so $(\alpha_i)_E$ is of the form (3.6). The r' -tuples $(\alpha_1, \dots, \alpha_{r'})^t$ and $(\beta_1, \dots, \beta_{r'})$ give us respectively two morphisms $\alpha \in \text{Hom}(M^{\oplus r'}, M(Y))$ and $\beta \in \text{Hom}(M(Y), M^{\oplus r'})$. By the construction of α and β , the matrix $(\text{mult}((\beta_j)_E \circ (\alpha_i)_E))_{1 \leq i, j \leq r}$ is an identity matrix. Then, by Lemma 3.1, $\beta_E \circ \alpha_E = ((\beta_j)_E \circ (\alpha_i)_E)_{1 \leq i, j \leq r} = 1_E$, where we note simply by 1 the identity morphism $((\delta_{i,j}))_{1 \leq i, j \leq r}$ in $\text{Hom}(M^{\oplus r}, M^{\oplus r})$. Let \mathfrak{X} be a disjoint union of r' copies of X , then $\text{Hom}(M(\mathfrak{X}), M(\mathfrak{X})) = \text{Hom}(M^{\oplus r'}, M^{\oplus r'})$. According to [4, Theorem 92.4] the variety \mathfrak{X} satisfies the nilpotence principle. By Lemma 2.3, there exist a positive integer n , such that $(\beta \circ \alpha)^n = 1$ (we apply Lemma 2.3 to the variety \mathfrak{X} and to the morphism $\beta \circ \alpha \in \text{Hom}(M(\mathfrak{X}), M(\mathfrak{X}))$). The morphisms α and $(\beta \circ \alpha)^{n-1} \circ \beta$ give the isomorphism between the motive $M^{\oplus r'}$ and a direct summand of $M(Y)$. Therefore $r' \geq r$ and then finally $r' = r$. \square

PROPOSITION 3.7. *Let D be a central division F -algebra of degree p^n with $n \geq 1$. Let X and Y be respectively the varieties $X(1, D)$ and $X(p^m, D)$, $0 \leq m < n$. Let E/F be a splitting field extension for the variety X , let T_1 and T_{p^m} be the tautological bundles of rank 1 and p^m on X_E and Y_E respectively. Then the subring of F -rational cycles in $\text{Ch}(X_E \times Y_E)$ is generated by the Chern classes of the vector bundle $T_1 \boxtimes (-T_{p^m})^\vee$ (we lift the bundles T_1 and T_{p^m} on $X_E \times Y_E$ and then take a product).*

Proof. Let Tav be the tautological vector bundle on X . The product $X \times Y$ considered over X (via the first projection) is isomorphic (as a scheme over X) to the Grassmann bundle $G_r(Tav)$ of r -dimensional subspaces in Tav (cf. [6, Proposition 4.3]), where $r = p^n - p^m$. Let T be the tautological r -dimensional vector bundle on $G_r(Tav)$. By [5, Example 14.6.6], the Chow ring $\text{Ch}(G_r(Tav))$ as an algebra over $\text{Ch}(X)$ is generated by Chern classes $c_0(T), c_1(T), \dots, c_r(T)$. By [7, Proposition 2.1.1], we have $\bar{\text{Ch}}(X) = \bar{\text{Ch}}^0(X) = \mathbb{Z} \cdot [X_E]$. Therefore the Chow ring $\bar{\text{Ch}}(X \times Y) \simeq \bar{\text{Ch}}(G_r(Tav))$ is generated (as a ring) by Chern classes $c_0(T_E), \dots, c_r(T_E)$. Since there exists an isomorphism (cf. [6, Proposition 4.3]): $T_E \simeq T_1 \boxtimes (-T_{p^m})^\vee$, we are done. \square

COROLLARY 3.8. *The motive with coefficients in \mathbb{F}_p of the variety $X(p^m, D)$ is decomposable for $p = 2, 1 < m < n$ and for $p > 2, 0 < m < n$. In these cases $M(X(1, D))(k)$ is a summand of $M(X(p^m, D))$ for $2 \leq k \leq p^n - p^m$.*

Proof. We use the notations: $X = X(1, D)$, $Y = X(p^m, D)$, $d = \dim(X(1, D)) = p^n - 1$, $r = p^n - p^m$. Let $E = F(X)$, then E/F is a splitting field extension for the variety X (and also for Y). Over the field E the algebra D becomes isomorphic to $\text{End}_E(V)$ for some E -vector space V of dimension $d + 1 = p^n$. We have $X_E \simeq \mathbb{P}^d(V)$ and $Y_E \simeq G_{p^m}(V)$. Let T_1 and T_{p^m} be the tautological bundles of rank 1 and p^m on X_E and Y_E respectively. We note by T the r -dimensional vector bundle $T_1 \boxtimes (-T_{p^m})^\vee$ on $X_E \times Y_E$. By Proposition

3.7, the ring $\overline{\text{Ch}}(X \times Y)$ is generated by Chern classes of the vector bundle T . Let $h = c_1(T_1) \in \text{Ch}^1(X_E)$ (then $-h$ is the hyperplane class in $\text{Ch}^1(X_E)$) and $c_i = c_i((-T_{p^m})^\vee) \in \text{Ch}^i(Y_E)$, $0 \leq i \leq r$. Then by [5, Remark 3.2.3(b)]

$$(3.9) \quad c_t(T) = c_t(T_1 \boxtimes (-T_{p^m})^\vee) = \sum_{i=0}^r (1 + (h \times 1)t)^{r-i} (1 \times c_i)t^i.$$

It follows from the conditions of the corollary that the binomial coefficients $\binom{p^n - p^m}{2}$, $\binom{p^n - p^m}{p^m - 1}$ are divisible by p and $\binom{p^n - p^m - 1}{p^m - 2} \equiv (-1)^{p^m - 2} \pmod{p}$. Therefore

$$\begin{aligned} c_1(T) &= (p^n - p^m)h \times 1 + 1 \times c_1 = 1 \times c_1, \\ c_2(T) &= \binom{p^n - p^m}{2} h^2 \times 1 + (p^n - p^m - 1)h \times c_1 + 1 \times c_2 = -h \times c_1 + 1 \times c_2, \\ c_{p^m - 1}(T) &= \binom{p^n - p^m}{p^m - 1} h^{p^m - 1} \times 1 + \binom{p^n - p^m - 1}{p^m - 2} h^{p^m - 2} \times c_1 + \dots = \\ &= (-1)^{p^m - 2} h^{p^m - 2} \times c_1 + \dots, \end{aligned}$$

where “...” stands for a linear combination of only those terms whose second factor has codimension > 1 . For the top Chern class we have:

$$c_r(T) = \sum_{i=0}^r h^{r-i} \times c_i.$$

For any integer $k \geq 2$ we define $\beta_k = c_r(T)c_{p^m - 1}(T)c_2(T)c_1(T)^{k-2} = (-h)^d \times c_1^k + \dots = [pt] \times c_1^k + \dots$, where “...” stands for a linear combination of only those terms whose second factor has codimension $> k$ and where $[pt]$ is the class of a rational point in $\text{Ch}(X_E)$. Let $f : X \times Y \rightarrow X$ be a projection onto the first factor. The cycle β_k is F -rational and $f_*(\beta_k) = c_1^k$. By [5, Example 14.6.6], the cycle c_1^k is non-zero for $2 \leq k \leq p^n - p^m$. Therefore $\dim_{\mathbb{F}_p} f_* \overline{\text{Ch}}_{\dim Y - k}(X \times Y) \geq 1$ for $2 \leq k \leq p^n - p^m$. The statement follows from Theorem 3.4. \square

REMARK 3.10. The Corollary 3.8 also gives us some information about the integral motive of the variety $X(p^m, D)$. Indeed, according to [12, Corollary 2.7] the decomposition of $M(X(p^m, D))$ with coefficients in \mathbb{F}_p lifts (and in a unique way) to the coefficients $\mathbb{Z}/p^N\mathbb{Z}$ for any $N \geq 2$. Then by [12, Theorem 2.16] it lifts to \mathbb{Z} (uniquely for $p = 2$ and $p = 3$ and non-uniquely for $p > 3$). See also Remark 4.14.

REMARK 3.11. Let l be an integer such that $0 < l < p^n$ and $\text{gcd}(l, p) = 1$. The complete decomposition of the motive $M(X(l, D))$ with coefficients in \mathbb{F}_p is described in [1, Proposition 2.4].

COROLLARY 3.12. *Let D be a central division F -algebra of p -primary index. Let K/F be a field extension, such that D_K is still division. Then the motive $(M_{1,D})_K$ is still indecomposable.*

Proof. We note by X and Y respectively the varieties $X(1, D)$ and $X(p, D)$. We note by M the motive $M(X)$. By [9, Theorem 3.8] the complete motivic decomposition of the variety Y consists of the motive $M_{1,D}$ and of the sum of motives M (we neglect the shifts in this proof). Suppose that the motive $(M_{1,D})_K$ is decomposable, then by the same theorem, M_K is a summand of $(M_{1,D})_K$. Therefore, the number of motives M_K in the complete motivic decomposition of Y_K is greater than the number of motives M in the complete motivic decomposition of Y . Let E/K be a splitting field extension for the algebra D . By Proposition 3.7, the subspace of K -rational cycles in $\text{Ch}(X_E \times Y_E)$ coincides with the subspace of F -rational cycles in $\text{Ch}(X_E \times Y_E)$. Therefore the Theorem 3.4 gives a contradiction. \square

4. COMPLETE MOTIVIC DECOMPOSITIONS

In the Corollary 3.8 we proved that the motive of the variety $X(p^m, D)$ is decomposable for $p = 2, 1 < m < n$ and for $p > 2, 0 < m < n$. Moreover, in these cases the Corollary 3.8 gives us a list of some motivic summands of the variety $X(p^m, D)$. By duality, we can extend this list. It happens, that in two small-dimensional cases $p = 3, m = 1, n = 2$ and $p = 2, m = 2, n = 3$ this is already a complete list of indecomposable motivic summands of the variety $X(p^m, D)$. Note that in general it is not true (see Example 4.8).

EXAMPLE 4.1. In this example we describe the complete motivic decomposition of $Y := X(3, D)$ for a division F -algebra D of degree 9. We note by X the variety $X(1, D)$ and by M the motive $M(X)$. Note that $\dim X = 8$ and $\dim Y = 18$.

By [9, Theorem 3.8], any indecomposable motivic summand of Y , besides the upper motive $M_{1,D}$, is some shift of M . By Corollary 3.8, the motives $M(2), M(3), M(4), M(5), M(6)$ and by duality $M(8), M(7)$ are direct summands of $M(Y)$. Suppose that there is at least one more motive $M(t)$ (for some integer $t \geq 0$) in the complete motivic decomposition of Y . Since by [9, Theorem 4.3] the variety Y is 3-incompressible, we have

$$\text{rk}_0 M(Y) = \text{rk}_0 M_{1,D} = \text{rk}_{\dim Y} M_{1,D} = \text{rk}_{\dim Y} M(Y) = 1.$$

It follows that $\text{rk}_0 M(t) = \text{rk}_{\dim Y} M(t) = 0$. We have

$$1 \leq t \leq \dim Y - \dim X - 1 = 9.$$

Since the decomposition of any of eight motives $M(2), M(3), \dots, M(8), M(t)$ into the sum of Tate motives over the splitting field contains a Tate motive $\mathbb{F}_3(9)$, then $\text{rk}_9 M_{1,D} \leq \text{rk}_9 M(Y) - 8$. According to [13, §2.5], we have $\text{rk}_9 M(Y) = 8$, therefore $\text{rk}_9 M_{1,D} = 0$.

By [8, Corollary 10.19], we have the following motivic decomposition of Y over the function field $L = F(Y)$:

$$(4.2) \quad M(Y)_L = \bigoplus_{i+j+k=3} M(X(i, C) \times X(j, C) \times X(k, C)),$$

where C is a central division L -algebra (of degree 3) Brauer-equivalent to D_L . Note that the triples $(3, 0, 0)$, $(0, 3, 0)$, $(0, 0, 3)$ correspond to three Tate motives \mathbb{F}_3 , $\mathbb{F}_3(9)$ and $\mathbb{F}_3(18)$. Let $\widetilde{M} = M(X(1, C))$, then by [8, Example 10.20], $M_L = \widetilde{M} \oplus \widetilde{M}(3) \oplus \widetilde{M}(6)$. It follows that the complete decomposition of M_L does not contain $\mathbb{F}_3(9)$. Therefore $\mathbb{F}_3(9)$ is a direct motivic summand of $(M_{1,D})_L$ and we have a contradiction with $\text{rk}_9 M_{1,D} = 0$.

The complete motivic decomposition of the variety $X(3, D)$ with coefficients in \mathbb{F}_3 is the following one:

$$(4.3) \quad M(X(3, D)) = M_{1,D} \oplus M(2) \oplus M(3) \oplus M(4) \oplus M(5) \oplus M(6) \oplus M(7) \oplus M(8).$$

EXAMPLE 4.4. Similarly, as in the previous example, we can find the complete motivic decomposition of $Y := X(4, D)$ for a division F -algebra D of degree 8. We note by M the motive $M(X(1, D))$.

By Corollary 3.8, the motives $M(2)$, $M(3)$, $M(4)$ and by duality $M(7)$, $M(6)$, $M(5)$ are direct summands of $M(Y)$. We have

$$M(X(4, D)) = M(2) \oplus \dots \oplus M(7) \oplus N$$

for some motive N . Assume that N is decomposable. Then by [9, Theorem 3.8], and Theorems 3.2, 3.3, the motive N has an indecomposable summand which is some shift of either $M_{0,D} = M$ or $M_{1,D} = M(X(2, D))$. But the second case is impossible because

$$70 = \binom{8}{4} = \text{rk } M(Y) < 6 \text{rk } M + \text{rk } M(X(2, D)) = 6 \cdot 8 + \binom{8}{2} = 76$$

(see [9, Example 2.18] for the computations of ranks). Therefore $M(t)$ is a summand of N for some integer t .

According to [8, Corollary 10.19], we can write the complete decomposition of N over the function field $L = F(Y)$:

$$N_L = \mathbb{F}_2 \oplus \widetilde{M}(1) \oplus M(X(2, C))(4) \oplus M(X(2, C))(8) \oplus \widetilde{M}(12) \oplus \mathbb{F}_2(16),$$

where C is a central division L -algebra (of degree 4) Brauer-equivalent to D_L and where $\widetilde{M} = M(X(1, C))$. It follows from this decomposition that the motive $M(t)_L = \widetilde{M}(t) \oplus \widetilde{M}(t+4)$ can not be a direct summand of N_L . We have a contradiction. Therefore the motive N is indecomposable and $N \simeq M_{2,D}$.

Now we can write the complete motivic decomposition of $X(4, D)$ with coefficients in \mathbb{F}_2 :

$$(4.5) \quad M(X(4, D)) = M_{2,D} \oplus M(2) \oplus M(3) \oplus M(4) \oplus M(5) \oplus M(6) \oplus M(7).$$

Let us consider the following class of generalized Severi-Brauer varieties.

DEFINITION 4.6. We say that the generalized Severi-Brauer variety $X(p^m, D)$ is of *type 0*, if the complete decomposition of $M(X(p^m, D))$ consists only of the upper motive $M_{m,D}$ and some (possibly zero) shifts of the motive $M_{0,D} = M(X(1, D))$.

For example, by [9, Theorem 3.8], the variety $X(p, D)$ is of this type. Let Y be a generalized Severi-Brauer $X(p^m, D)$ variety of type 0. By Theorem 3.4, the subspace of F -rational cycles in $\text{Ch}(X_E \times Y_E)$ describes the complete motivic decomposition of Y , where $X = X(1, D)$, $E = F(X)$. Note that the structure of the ring $\text{Ch}(X_E \times Y_E) = \text{Ch}(X_E) \times \text{Ch}(Y_E)$ is well-known (cf. [5, § 14]) and by Proposition 3.7 we can compute the subring $\bar{\text{Ch}}(X \times Y) \subset \text{Ch}(X_E \times Y_E)$. Therefore we can say that the complete motivic decomposition of any generalized Severi-Brauer variety $X(p^m, D)$ of type 0 can be “theoretically” found in a finite time using computer.

REMARK 4.7. We do not possess a single example of a variety $X(p^m, D)$, which is not of type 0. Therefore, it may happen that the generalized Severi-Brauer variety $X(p^m, D)$ is always of type 0 (for any division F -algebra D of degree p^n and for any integer m , $0 \leq m \leq n$). Note that if this is true, then Conjecture 1.1 holds (one can follow the lines of the proof of Corollary 3.12).

EXAMPLE 4.8. Let D be a central division F -algebra of degree 27. In this example we find complete motivic decomposition of the variety $Y = X(3, D)$, which is of type 0. We take the same notations as in the proof of Corollary 3.8: $X = X(1, D)$, $E = F(X)$, $T = T_1 \boxtimes (-T_3)^\vee$, where T_1 and T_3 are the tautological bundles of rank 1 and 3 on X_E and Y_E respectively (the vector bundle T is of the rank 24). We note also by V_* the graded \mathbb{F}_3 -vector space $f_* \bar{\text{Ch}}_{\dim Y - *}(X \times Y)$, where f is a projection onto the second factor.

By Theorem 3.4, for any integer k the number of motives $M(k)$ in the complete motivic decomposition of Y is equal to $\dim_{\mathbb{F}_3} V_k$, where $M = M(X)$. By duality, this number is also equal to the number of motives $M(\dim Y - \dim X - k) = M(46 - k)$ in the same decomposition. Therefore the vector space $V_{\leq 23}$ describes the complete motivic decomposition of Y .

Let $h = c_1(T_1) \in \text{Ch}^1(X_E)$ and $c_i = c_i((-T_3)^\vee) \in \text{Ch}^i(Y_E)$, $0 \leq i \leq 24$. Using the formula 3.9 we can compute the following Chern classes of the vector bundle T :

$$\begin{aligned} c_1(T) &= 1 \times c_1, & c_2(T) &= -h \times c_1 + 1 \times c_2, \\ c_7(T) &= 1 \times c_7, & c_8(T) &= -h \times c_7 + 1 \times c_8, \\ c_{24}(T) &= h^{24} \times 1 + \sum_{i=1}^{24} h^{24-i} \times c_i. \end{aligned}$$

We have:

$$\begin{aligned} c_1^2 &= f_*(c_{24}(T)(c_2(T))^2) \in V_2, & c_1 c_7 &= f_*(c_{24}(T)c_2(T)c_8(T)) \in V_8, \\ c_7^2 &= f_*(c_{24}(T)(c_8(T))^2) \in V_{14}. \end{aligned}$$

Also we have the following property:

$$(4.9) \quad x \in V_* \Rightarrow (x c_1 \in V_{*+1} \quad \text{and} \quad x c_7 \in V_{*+7}).$$

Indeed, if $x \in V_*$, then $x = f_*(y)$ for some $y \in \bar{\text{Ch}}_{\dim Y - *}(X_E \times Y_E)$. Therefore $x c_1 = f_*(y \cdot c_1(T)) \in V_{*+1}$ and $x c_7 = f_*(y \cdot c_7(T)) \in V_{*+7}$.

This property gives us the following elements in V_i :

$$(4.10) \quad \begin{array}{ll} c_1^i & \text{if } 2 \leq i \leq 7, \\ c_1^i, c_1^{i-7} c_7 & \text{if } 8 \leq i \leq 13, \\ c_1^i, c_1^{i-7} c_7, c_1^{i-14} c_7^2 & \text{if } 14 \leq i \leq 20, \\ c_1^i, c_1^{i-7} c_7, c_1^{i-14} c_7^2, c_1^{i-21} c_7^3 & \text{if } 21 \leq i \leq 23. \end{array}$$

We also define a sequence $b_i, i \in \mathbb{Z}$:

$$b_i = \begin{cases} 0 & \text{if } i < 2 \\ 1 & \text{if } 2 \leq i \leq 7 \\ 2 & \text{if } 8 \leq i \leq 13 \\ 3 & \text{if } 14 \leq i \leq 20 \\ 4 & \text{if } 21 \leq i \leq 23 \\ b_{46-i} & \text{if } i > 23. \end{cases}$$

Note that for any $i \leq 23$ the number of elements lying in V_i from the list 4.10 is equal to b_i .

We are going to show that all elements from the list 4.10 are linearly independent (to apply then Theorem 3.4). The \mathbb{F}_3 -vector space V_* is a subspace of $\text{Ch}^*(Y_E)$. We note $\tilde{c}_i = c_i(T_3^\vee), i = 1, 2, 3$, where T_3 is the tautological bundle of rank 3 on Y_E . According to [5, Example 14.6.6] the graded ring $\text{Ch}^*(Y_E)$ is generated by Chern classes $\tilde{c}_1, \tilde{c}_2, \tilde{c}_3, c_1, \dots, c_{24}$ modulo the homogeneous relations

$$(4.11) \quad c_r + c_{r-1}\tilde{c}_1 + c_{r-2}\tilde{c}_2 + c_{r-3}\tilde{c}_3 = 0 \quad \text{for } r = 1, \dots, 27,$$

where $c_i = 0$ for $i < 0$ or $i > 24$. It follows that the graded ring $\text{Ch}^*(Y_E)$ is generated by only three elements $\tilde{c}_1, \tilde{c}_2, \tilde{c}_3$ modulo some homogeneous relations of degree greater than 23. Therefore we have an isomorphism:

$$\text{Ch}^{*\leq 23}(Y_E) \simeq \mathbb{F}_3[\tilde{c}_1, \tilde{c}_2, \tilde{c}_3]_{\leq 23}.$$

Using relations 4.11 we can compute that $c_1 = -\tilde{c}_1$ and $c_7 = -\tilde{c}_1^7 + \tilde{c}_1^4 \tilde{c}_3 - \tilde{c}_1^3 \tilde{c}_2^2 + \tilde{c}_1 \tilde{c}_2^3$.

Now we consider the elements from the list 4.10 as polynomials in $\mathbb{F}_3[\tilde{c}_1, \tilde{c}_2, \tilde{c}_3]$. To show that all of them are linearly independent, it suffices to check this for four elements $c_1^{21}, c_1^{14} c_7, c_1^7 c_7^2, c_7^3$ (they are in our list) from V_{21} . Since the polynomial ring $\mathbb{F}_3[\tilde{c}_1, \tilde{c}_2, \tilde{c}_3]$ is factorial and c_7 is not divisible by \tilde{c}_1^2 , then for any $\alpha, \beta, \gamma, \delta \in \mathbb{F}_3$ we have

$$\alpha c_1^{21} + \beta c_1^{14} c_7 + \gamma c_1^7 c_7^2 + \delta c_7^3 = 0 \implies \alpha = \beta = \gamma = \delta = 0.$$

Since all elements from the list 4.10 are linearly independent, then $\dim_{\mathbb{F}_3} V_i \geq b_i$ for $i \leq 23$. Therefore for any integer i the motive $M^{\oplus b_i}(i)$ is a direct summand of $M(Y)$. Indeed, the statement follows from Theorem 3.4 for $i \leq 23$ and by duality it is also true for $i > 23$. We have

$$(4.12) \quad M(Y) = \bigoplus_{i \in \mathbb{Z}} M^{\oplus b_i}(i) \oplus N$$

for some motive N over F .

Now we want to understand the complete decomposition of N . Let L be a function field of the variety Y and C be a central division L -algebra (of degree

9) Brauer-equivalent to D_L . Using the motivic decomposition similar to the decomposition 4.2 from Example 4.1, we can show that the complete decomposition of $M(Y)_L$ contains three indecomposable motives: $M_{1,C}$, $M_{1,C}(27)$, $M_{1,C}(54)$. Moreover any other summand in the complete motivic decomposition of $M(Y)_L$ is a shift of the motive $\widetilde{M} := M(X(1, C))$. We know that $M_L = \widetilde{M} \oplus \widetilde{M}(9) \oplus \widetilde{M}(18)$. It follows that $N_L = M_{1,C} \oplus M_{1,C}(27) \oplus M_{1,C}(54) \oplus N'$ for some motive N' over L and N' is a sum of shifts of the motive \widetilde{M} . Note that if $M(k)$ is direct summand of N for some integer k , then $M_L(k)$ is a direct summand of N' .

Let S be a direct summand of the motive of a geometrically split variety. We write $P(S, t)$ for the Poincaré polynomial of S :

$$P(S, t) = \sum_{i \geq 0} (\text{rk}_i S) \cdot t^i.$$

Let us find the Poincaré polynomial of the motive N' . We have

$$P(N', t) = P(M(Y), t) - (1 + t^{27} + t^{54})P(M_{1,C}, t) - \sum_{i \geq 0} b_i t^i P(M, t).$$

Using the following formulas

$$P(M(Y), t) = \frac{(1 - t^{27})(1 - t^{26})(1 - t^{25})}{(1 - t)(1 - t^2)(1 - t^3)}, \text{ (according to [13, §2.5]),}$$

$$P(M, t) = \frac{1 - t^{27}}{1 - t} = \sum_{i=0}^{26} t^i,$$

$$P(M_{1,C}, t) = t^6 + t^{12} + \sum_{i=0}^{26} t^i, \text{ (by Example 4.3) ,}$$

we can compute $P(N', t)$. Since N' is a sum of shifts of the motive \widetilde{M} then $P(N', t)$ is divisible by $P(\widetilde{M}, t) = (1 - t^9)/(1 - t) = 1 + t + \dots + t^8$. Let $Q(t)$ be a quotient of these two polynomials. After computations, we have

$$Q(t) = t^7 + t^{13} + t^{16} + t^{18} + t^{19} + t^{20} + t^{22} + t^{24} + t^{26} + t^{28} + t^{29} + t^{30} + t^{34} + t^{35} + t^{36} + t^{38} + t^{40} + t^{42} + t^{44} + t^{45} + t^{46} + t^{48} + t^{51} + t^{57}.$$

Now if $M(k)$ is direct summand of N for some integer k , then $M_L(k) = \widetilde{M}(k) \oplus \widetilde{M}(k + 9) \oplus \widetilde{M}(k + 18)$ is a direct summand of N' . Therefore in this case the decomposition of $Q(t)$ contains $t^k + t^{k+9} + t^{k+18} = P(M_L(k), t)/P(\widetilde{M}, t)$. Only two values $k = 20$ and $k = 26$ satisfy this condition. Note that if complete decomposition of the motive N contains $M(20)$ then by duality it contains also $M(26)$. It follows that the question of the complete motivic decomposition of Y reduces to the question either $\dim_{\mathbb{F}_3} V_{20} = 3$ or $\dim_{\mathbb{F}_3} V_{20} = 4$? Let us show that we are in the second case.

Consider the following cycle e from V_{20}

$$e = f_*(c_2^{11}(-T)c_3^8(-T)) = -\tilde{c}_1^{17}\tilde{c}_3 + \tilde{c}_1^{16}\tilde{c}_2^2 - \tilde{c}_1^{14}\tilde{c}_2^3 - \tilde{c}_1^{14}\tilde{c}_3^2 - \tilde{c}_1^{13}\tilde{c}_2^2\tilde{c}_3 - \tilde{c}_1^{12}\tilde{c}_2^4 + \tilde{c}_1^{11}\tilde{c}_2^3\tilde{c}_3 - \tilde{c}_1^{11}\tilde{c}_3^3 - \tilde{c}_1^{10}\tilde{c}_2^5 - \tilde{c}_1^2\tilde{c}_2^9,$$

where $c_2(-T) = -h\tilde{c}_1 + \tilde{c}_2$, $c_3(-T) = h^3 + h^2\tilde{c}_1 + h\tilde{c}_2 + \tilde{c}_3$. The cycle e as a polynomial in $\mathbb{F}_3[\tilde{c}_1, \tilde{c}_2, \tilde{c}_3]$ is not divisible by \tilde{c}_1^3 . It follows that the cycle e could not be a linear combination of three cycles c_1^{20} , $c_1^{13}c_7$, $c_1^6c_7^2$ from the list 4.10. Therefore $\dim_{\mathbb{F}_3} V_{20} = 4$.

Consider a sequence $(a_i)_{i \in \mathbb{Z}}$ defined by

$$a_i = \begin{cases} b_i + 1 & \text{if } i = 20 \text{ or } i = 26 \\ b_i & \text{else.} \end{cases}$$

The complete motivic decomposition of the variety Y is the following

$$(4.13) \quad M(Y) = \bigoplus_{i \in \mathbb{Z}} M^{\oplus a_i} \oplus M_{1,D}.$$

REMARK 4.14. We have the same decompositions as (4.3), (4.5) and (4.13) for the motives with the integral coefficients. To show this one can apply [12, Corollary 2.7] and then [12, Theorem 2.16].

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