Discrete Optimization Stories

There are a number of very good surveys of the history of combinatorial optimization (briefly CO). I want to recommend to the reader two outstanding articles: [5] covers the area until 1960 and [2] the history of integer programming in the last ~ 50 years. And there is the encyclopedic 3-volume book [6] which is an unsurpassable source book for the historical development of CO. Nevertheless, the articles in this section shed some new light on certain historical aspects of CO.

The original plan of this book included further remarkable CO-stories. They had to be abandoned for space reasons. But I want to mention two of them in this introduction because there are good sources available where the details can be found.

Let me begin with a most astonishing discovery. One of the first algorithms of CO I heard about was the Hungarian method which has been viewed by many as a prototype of algorithm design and efficiency. Harold Kuhn presented it in 1955 in [3]. Having used ideas and results of J. Egerváry and D. Kőnig, he gave his algorithm (generously) the name Hungarian method. In 2004 the journal Naval Research Logistics Quarterly (briefly NRL) established a new "best paper award" to recognize outstanding research published in NRL. [3] was selected as the best paper published since 1954 in NRL, and A. Frank [2] wrote a moving paper about "Kuhn's Hungarian Method" in NRL. In 2005 A. Frank organized a conference in Budapest entitled "Celebration Day of the 50th Anniversary of the Hungarian Method" at which I highlighted the role the Hungarian algorithm has played in practical applications such as vehicle scheduling. Soon thereafter, on March 9, 2006 I received an e-mail from Harold Kuhn that started as follows:

> `Dear Friends:`
>
> `As participants in the 50th Birthday celebration of the Hungarian Method, you should be among the first to know that Jacobi discovered an algorithm that includes both Koenig's Theorem and the Egervary step. I was told about Jacobi's paper by Francois Ollivier who has a website with the original papers and French and English translations. They were published in Latin after his death and so the work was done prior to 1851!!!`

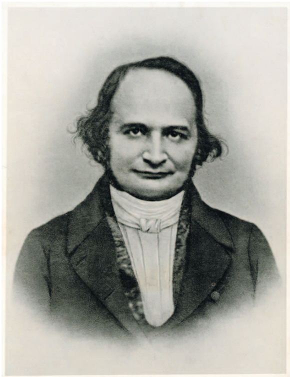

Figure 1: Carl G. J. Jacobi
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Figure 2: Jacobi's grave
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What a surprise! The Hungarian method had appeared for the first time in a paper, written in Latin, attempting to establish a bound on the degree of a system of partial differential equations and which was only published posthumously in Jacobi's collected works. The original manuscript can be found in the "Jacobi Nachlass" of the BBAW archive in Berlin. I will not go into the details of the story since Harold Kuhn has written up all the circumstances in his recent article [4], where one can find all the relevant references. I just want to remark that the Jacobi mentioned is Carl Gustav Jacob Jacobi, see Fig. 1, after whom the Jacobi matrix is named. Jacobi was born in Potsdam in 1804, became Professor in Königsberg in 1826, moved to Berlin in 1843, and died in 1851. Jacobi has an "honorary grave" (Ehrengrab) on the "Friedhof der Berliner Dreifaltigkeitsgemeinde" in Berlin, see Fig. 2.

The second story is of completely different nature. It is about mathematics done under extreme circumstances. I just want to quote pieces of a paper [7] written by Paul Turán, one of the great Hungarian figures of combinatorics, about some of his experiences in World War II.

In 1940 Turán had to work on railway building in a labor camp in Transylvania and proved what we call Turàns theorem today. In his words:

> *... I immediately felt that here was the problem appropriate to the circumstances. I cannot properly describe my feelings during the next few days. The pleasure of dealing with a quite unusual type of problem, the beauty of it, the gradual nearing of the solution, and finally the complete solution made these days really ecstatic. The feeling of some intellectual freedom and being, to a certain extent, spiritually free of oppression only added to this ecstasy.*

The second experience I want to mention is about Turán's discovery of the crossing number. He writes:

> *In July 1944 the danger of deportation was real in Budapest, and a reality outside Budapest. We worked near Budapest, in a brick factory. There were some kilns where the bricks were made and some open storage yards where the bricks were stored. All the kilns were connected by rail with all the storage yards. The bricks were carried on small wheeled trucks to the storage yards. All we had to do was to put the bricks on the trucks at the kilns, push the trucks to the storage yards, and unload them there. We had a reasonable piece rate for the trucks, and the work itself was not difficult; the trouble was only at the crossings. The trucks generally jumped the rails there, and the bricks fell out of them; in short this caused a lot of trouble and loss of time which was rather precious to all of us (for reasons not to be discussed here). We were all sweating and cursing at such occasions, I too; but nolens-volens the idea occurred to me that this loss of time could have been minimized if the number of crossings of the rails had been minimized. But what is the minimum number of crossings?*

Let us all hope that mathematics discoveries will never again have to be made under such circumstances.

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References

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