Stable Maps of Curves

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Abstract. Let $h: X \to Y$ be a finite morphism of smooth connected complete curves over $\mathbf{C}_p$. We show h extends to a finite morphism between semi-stable models of X and Y.

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Let p be a prime. It is known that if C is a smooth proper curve over a complete subfield K of C_p, there exists a finite extension L of K in C_p and a model of the base extension of C to L over the ring of integers, R_L, of L whose reduction modulo the maximal ideal has at worst ordinary double points as singularities. In fact, if $g(C)$, the genus of C, is at least 2 or $g(C) = 1$ and C has a model with good reduction, there is a minimal such model, which is called the stable model. Indeed, if L' is any complete extension of L in C_p, the base extension of a stable model over R_L is the stable model over $R_{L'}$.

Liu and Lorenzini showed [L-L; Proposition 4.4(a)] that a finite morphism of curves extends to a morphism of stable models, but the extension is not in general finite. E.g., Edixhoven has show that the natural map from $X_0(p^2)$ to $X_0(p)$ does not in general extend to a finite morphism of stable models [E] (see also [C-M]). However, we show,

Theorem. *Suppose $h: X \to Y$ is a finite morphism of smooth connected complete curves over C_p. Then there are semi-stable models $\mathcal{X}$ and $\mathcal{Y}$ of X and Y over the ring of integers of C_p such that h extends to a finite morphism from $\mathcal{X}$ to $\mathcal{Y}$.*

(We work over C_p to avoid having to worry about base extensions and because reduced affinoids over C_p have reduced reductions.)

In fact, when X has a stable model (i.e., $g(X) \geq 2$ or $g(X) = 1$ and X has good reduction) and either X/Y is Galois, or the model has irreducible reduction and Y has a stable model, one can take $\mathcal{X}$ to be the stable model for X. In the latter case $\mathcal{Y}$ will be the stable model for Y, which, a fortiori, will have irreducible reduction.

Abbes has informed us that this result also follows from results of Raynaud (in particular Proposition 5 of [R] (and its corollary)).

We say such a morphism is semi-stable, and stable if it is the minimal object in the category of semi-stable morphisms from X to Y (which may not exist).

Terminology and Notation

If Z is a rigid space $A(Z)$ will denote the ring of analytic functions on Z and $A^0(Z)$ the subring of functions whose spectral norm is bounded by 1.

If X is an affinoid over C_p, $\overline{X} =$ Spec$A^0(X)/\mathfrak{m}A^0(X)$, where $\mathfrak{m}$ is the maximal ideal of R_p and if $x \in \overline{X}(\bar{F}_p)$, R_x will denote the corresponding residue class in X. (Residue classes are also called formal fibers.)

By a regular singular point on a curve we mean a singular point which is an ordinary double point. If C is a curve, let $S(C)$ denote the set of irregular singular points on C.

1. Wide Opens

In this section, we review and extend the results on wide open spaces discussed in [RLC].

A (smooth one-dimensional) wide open is a rigid space conformal to $C-D$ where C is a smooth complete curve and D is a finite disjoint union of affinoid disks in C, which contains at least one in each connected component. A wide open disk is the complement of one affinoid disk in P^1 (it is conformal to $B(0,1)$) and a wide open annulus is conformal to the complement of two disjoint such disks (it is conformal to $A(r,1)$ where $r \in |C_p|$, $0 < r < 1$).

An underlying affinoid Z of a wide open W is an affinoid subdomain Z of W such that $W \backslash Z$ is a finite disjoint union of annuli none of which is contained in an affinoid subdomain of W. An end of W is an element of the inverse limit of of the set of connected components of $W \backslash Z$ where Z ranges over subaffinoids of W.

We slightly modify the definition of basic wide open given in [RLC] and say a wide open W is basic if it has an underlying affinoid Z such that $\overline{Z}$ is irreducible and has at worst regular singular points.

Suppose X is a smooth one dimensional affinoid over C_p and $x \in \overline{X}$. Because $A^0(R_x)$ is the completion of $A^0(X)$ at x, we have,

Lemma 1.1. *Then x is a smooth point of $\overline{X}$ if and only if R_x is a wide open disk and a regular singular point if and only if R_x is a wide open annulus.*

We have the following generalization of Proposition 3.3(ii) of [RLC],

Lemma 1.2. *The residue class, R_x, is a connected wide open and its ends can naturally be put in 1-1 correspondence to the branches of $\overline{X}$ through x.*

Proof. That R_x is connected is a consequence of Satz 6 of [B].

Theorem A-1 of [pAI] and its proof naturally generalizes to SEMI-DAGGER ALGEBRAS. These are quotients of the rings of series $\sum_{I,J} a_{I,J} x^I y^J$ in $K[x_1, \ldots, x_n, y_1, \ldots, y_m]$, where K is a complete non-Archimedean valued field, such that there exists $r_{\in R} > 1$ so that

$$\lim_{s(I,J)\to\infty} |a_{I,J}| r^{s(J)} = 0,$$

where I and J range over $Z^n_{\geq 0}$ and $Z^m_{\geq 0}$ and $s(M)$, where M is a multi-index, is the sum of its entries. What this implies in our context is that if R is an affinoid over C_p whose reduction is equal to the normalization of $\overline{X}$ and S is the set of points of $\overline{R}$ above T, the singular points of $\overline{X}$, then the rings

$$\lim_{\substack{\rightarrow \\ M}} A(R\backslash M) \quad \text{and} \quad \lim_{\substack{\rightarrow \\ N}} A(X\backslash N)$$

are isomorphic, where M ranges over the subaffinoids of $\bigcup_{s\in S} R_s$ and N ranges over the subaffinoids of $\bigcup_{x\in T} R_x$. Since $\bigcup_{s\in S} R_s$ is a union of wide open disks which correspond to the set, B, of branches of $\overline{X}$ through points in T, this, in turn, implies that there exists a subaffinoid N of $\bigcup_{x\in T} R_x$ such that $\bigcup_{x\in \overline{T}} R_x - N$ is a finite union of wide open annuli which correspond to the elements of B. One can now glue affinoid disks to $\bigcup_{x\in T} R_x$ to make a smooth complete curve, using [B; Satz 6.1] and the direct image theorem of [K], as in the proof of Proposition 3.3 of [RLC]. The result follows. ∎

It follows from Lemmas 3.1 and 3.2 of [RLC] that,

Lemma 1.3. *if A and B are disjoint wide open annuli or disjoint affinoids in a smooth curve C over C_p, then A is disconnected from B in C.*

If W is a wide open space

$$H^1_{DR}(W) = \Omega^1_W / \mathrm{d}A(W),$$

where Ω^1_W is the $A(W)$-module of rigid analytic differentials on W. It follows from Theorem 4.2 of [RLC] that $H^1_{DR}(W)$ is finite dimensional over C_p.

Lemma 1.4. *Suppose $f\colon W \to V$ is a finite morphism of wide opens. Then, if W is a disk or annulus, the same is true for V.*

Proof. Suppose W is a disk and f has degree d. Then V has only one end. Suppose ω is a differential on V. Then $f^*\omega = \mathrm{d}g$ for some function $g \in A(W)$ since $\dim H^1_{DR}(W) = 0$, in this case. Hence $\omega = \mathrm{d}\ \mathrm{Tr}\,(g/d)$. Thus $H^1_{DR}(V) = 0$. Let C be a proper curve obtained by glueing a wide open disk D to V along the end, as in the proof of Proposition 3.3 of [RLC]. From the Meyer-Vietoris long exact sequence, we see that C has genus zero and as $B := D\backslash V$ is an affinoid disk $V = C\backslash B$ is a wide open disk.

The argument in the case where W is an annulus is similar, except one has to use residues. ∎

Proposition 1.5. *Suppose $f\colon X \to Y$ is a finite map of smooth one dimensional affinoids over C_p. Then, if the reduction of X has only regular singular points, the same is true of Y.*

Proof. We know the map f induces a finite morphism $\bar{f}\colon \bar{X} \to \bar{Y}$. Let y be a point of $\bar{Y}$. Let $x \in \bar{X}$ such that $\bar{f}(x) = y$. Then f restricts to a finite morphism $R_x \to R_y$. But R_x is a disk or annulus. It follows from Lemma 1.2 that R_y is a wide open and hence by Lemma 1.4 is a disk or annulus, as well. Hence y is either smooth or regular singular by Lemma 1.1. ∎

This implies the well known result that if $h\colon X \to Y$ is a finite morphism of curves and X has good reduction so does Y. In fact, it implies the result of Lorenzini-Liu, [L-L; Corollary 4.10], that, in this case if $g(Y) \geq 1$, h extends to a finite morphism between the unique models of good reduction. It also implies that if X has a stable model with irreducible reduction, so does Y.

Lemma 1.6. *Suppose $\phi\colon X \to Y$ is a non-constant rigid morphism of smooth one dimension affinoids over C_p. Suppose G is a finite group acting on X such that $\phi^\sigma = \phi$ for $\sigma \in G$ and $\overline{X} = \bigcup_{\sigma \subset G} V^\sigma$ where V is an irreducible component of $\overline{X}$. Then ϕ surjects onto an affinoid subdomain of Y.*

Proof. Let $x \in X$. Because ϕ is non-constant we can find an element f of $A(Y)$ such that $f(\phi(x)) = 0$ and $|\phi^* f|_X = 1$. We can and will replace Y with the affinoid subdomain $\{y \in Y\colon |f(y)| \leq 1\}$. Then $\overline{\phi}|_V$ is non-constant. Since $\overline{\phi X} = \overline{\phi} V$ and V is irreducible, $\overline{\phi}$ factors through the inclusion of an irreducible component S of $\overline{Y}$. Let S^0 be the complement in S of the other irreducible components of $\overline{Y}$. Then, $Z = \mathrm{red}^{-1}\mathrm{S}^0$ is an affinoid subdomain of Y whose reduction is S^0. Let X' be the affinoid subdomain of X, $\phi^{-1}Z$. This is just X minus a finite number of residue classes stable under the action of G so its reduction is the union of the G-conjugates of $V' = V \backslash \overline{\phi}^{-1}(S \backslash S^0)$ which is irreducible. Suppose $s \in \overline{\phi X} \cap S^0$. We claim that $R_s \subset \phi(X)$.

Suppose $y_0 \in R_s \backslash \phi(X)$. Since the class group of Z is torsion, there exists an $h \in A(Z)$ such that y_0 is the only zero of h. Because $\overline{Z}$ is irreducible, we can also suppose $|h|_Z = 1$. Since $h(y_0) = 0$, it follows that $|h(y)| < 1$ for $y \in R_s$. Let $g = \phi^* h \in A^0(X')$. If $y_0 \notin \phi(X)$, $1/g \in A(X')$ but $|1/g|_{X'} = |c| > 1$, for some $c \in C_p$, since s is in the image of $\overline{\phi}_{\overline{X}'}$. However,

$$|g(1/cg)|_{X'} = |c^{-1}| < 1 = |g|_{X'} |(1/cg)|_{X'}.$$

This implies $\overline{g}_V = 0$ or $\overline{(1/cg)}_V = 0$, but as $X' = \bigcup_{\sigma \in G} V'^\sigma$, this implies the contradiction that $\overline{g} = 0$ or $\overline{(1/cg)} = 0$.

We will finish the proof by showing $X = X'$.

Let Y' be the affinoid obtained by glueing in disks to $\mathrm{red}^{-1}\mathrm{S}$ at the ends of the wide open $\mathrm{red}^{-1}\mathrm{S} \backslash \mathrm{S}^0$ corresponding to irreducible components of $\overline{Y}$ distinct from S. The reduction of this affinoid is naturally isomorphic to S. Then as ϕ factors through the inclusion of $\mathrm{red}^{-1}\mathrm{S}$ in Y we naturally obtain a morphism $\phi'\colon X \to Y'$. Since by construction, for each point $s \in S \backslash S^0$, there is a point in the residue class of Y' not in the image of X the above argument implies no point in this residue class is in the image of X and so $\phi'(X)$ in the reduction inverse in Y' of S^0. As this latter is naturally isomorphic to Z, $X = X'$. ∎

Suppose W is a wide open annulus. If $\sigma: W \to W$ is a rigid analytic morphism, define $\rho(\sigma)$ by

$$\rho(\sigma)\ \mathrm{Res}\omega = \ \mathrm{Res}\,\sigma^*\omega.$$

The restriction of ρ to the group of rigid analytic automorphisms of W is a homomorphism from $Aut(W)$ onto $\{\pm 1\}$. We say σ is ORIENTATION PRESERVING if $\rho(\sigma) = 1$.

Lemma 1.7. *Suppose G is a finite group of rigid automorphisms of the wide open annulus $W = A(r,1)$ of order m. Then there is a rigid morphism $\phi: W \to V$ of degree m such that $A(W)^G = \phi^* A(V)$, where $V = A(r^m, 1)$ if G is orientation preserving and $V = A(B(0,1))$ if not.*

Proof. First, if $\sigma \in G$

$$\sigma^* T = c_\sigma T^{\rho(\sigma)} h_\sigma(T),$$

where $h_\sigma(T) \in A(W)$, $|h_\sigma(t) - 1| < 1$, for $t \in W$, and $c_\sigma \in C_p$,

$$|c_\sigma| = \begin{cases} 1 & \text{if } \rho(\sigma) = 1 \\ r & \text{if } \rho(\sigma) = -1 \end{cases}.$$

Let $G^o = \mathrm{Ker}\,\rho$ and $n = |G^o|$. Let $S = \prod_{\tau \in G^o} \tau^* T$. Then

$$S(T) = T^n g(T),$$

where $|g(t)| = 1$. Let $\alpha: W \to A(r^n, 1)$, be the map

$$t \mapsto S(t).$$

It is easy to see this map has degree n and $R := \alpha^* A(A(r^n, 1)) \subseteq A(W)^G$. In particular, R is an integral domain and its fraction field is K^G where K is the fraction field of $A(W)$. Since, R and $A(W)$ are Dedekind domains, it follows that $R = A(W)^{G_0}$. If G is orientation preserving, $G = G^o$ and taking $\phi = \alpha$ completes the proof, in this case.

Suppose now G is not orientation preserving. Then G/G^o has order 2. Using, the result of the last paragraph we can replace W with $A(r^n, 1)$ and assume G^o is trivial. Let $G = \{1, \sigma\}$ and

$$U(T) = T + \sigma^* T = T + c_\sigma T^{-1} h_\sigma(T).$$

Now, if we define $\phi: W \to B(0,1)$ to be the morphism

$$t \mapsto U(t),$$

we can apply the same argument, as above, to complete the proof. ∎

REMARK. *One can show:*

PROPOSITION. *Suppose p is odd G is a finite group of automorphisms of $A(r,R)$. Then there is a natural homomorphsm of G into $\mathrm{Aut}\, G_m^{\overline{F}_p}$ whose kernel is the unique p-Sylow subgroup of G^o. Moreover, the exact sequence,*

$$1 \to G^o \to G \to G/G^o \to 1,$$

splits.

For example: Suppose $p = 3$, $1 > |r| > |27|$ and $V = A(r,1)$. Let s be the parameter on A^1. Then the integral closure of $A(V)$ in the splitting field of $X^3 + sX = s$ over $K(V)$ is the ring of analytic functions on an annulus W which is an étale Galois cover of V. If G is the Galois group, $G = G^o \cong S_3$.

2. Semi-stable Coverings

A SEMI-STABLE COVERING of a curve C is a finite admissible covering $\mathcal{D}$ of C by connected wide opens such that
(i) if $U \neq V \in \mathcal{D}$, $U \cap V$ is a finite collection of disjoint wide open annuli,
(ii) if $T, U, V \in \mathcal{D}$ are pairwise distinct, $T \cap U \cap V = \emptyset$.
(iii) for $U \in \mathcal{D}$, if

$$U^u = U \backslash \big(\bigcup_{\substack{V \in \mathcal{D} \\ V \neq U}} V \big),$$

U^u is a non-empty affinoid whose reduction is irreducible and has at worst regular singular points.

In particular, if $U \in \mathcal{D}$, U is a basic wide open and U^u is an underlying affinoid of U. We let $E(U)$ denote the set of connected components of $U \backslash U^u$. These are all wide open annuli.

PROPOSITION 2.1. *Semi-stable models of C whose reductions have at least two components correspond to semi-stable covers of C.*

Proof. Suppose $\mathcal{C}$ is a semi-stable model for C whose reduction $\overline{\mathcal{C}}$ has at least two components. Let $I_{\mathcal{C}}$ denote the set of irreducible components of $\overline{\mathcal{C}}$. If $Z \in I_{\mathcal{C}}$ let $Z^0 = Z \backslash \bigcup_{\substack{A \in I_{\mathcal{C}} \\ A \neq Z}} A$ and $W_Z := \mathrm{red}^{-1}\mathrm{Z}$. As every singular point of $\overline{\mathcal{C}}$ is regular it follows from Lemma 1.2 that W_Z is a basic wide open with underlying affinoid $\mathrm{red}^{-1}\mathrm{Z}^0$ and $\{W_Z \colon Z \in I_{\mathcal{C}}\}$ is a semi-stable cover.

Conversely, suppose $\mathcal{D}$ is a semi-stable cover of C. For $U, V \in \mathcal{D}$, let $Z_U = \mathrm{Spf}\, A^0(Z)$ and $Z_{U,V} = \mathrm{Spf}\,(U \cap V)$. Then the formal schemes Z_U glue by means of the glueing data

$$Z_{U,V} \to Z_U \coprod Z_V$$

into a model $\mathcal{S}_{\mathcal{D}}$ of C. ∎

If we have semi-stable coverings $\mathcal{D}_X$ and $\mathcal{D}_Y$ such that for every $W, V \in \mathcal{D}_X$, $h(W) \in \mathcal{D}_Y$ and there exist $W', V' \in \mathcal{D}_X$ such that $h(W) \cap h(V) = h(W' \cap V')$, then f extends to a finite morphism from $\mathcal{S}_{\mathcal{D}_X}$ to $\mathcal{S}_{\mathcal{D}_Y}$. We say h induces a finite morphism of semi-stable covers from $\mathcal{D}_X$ to $\mathcal{D}_Y$.

3. Proof of Theorem

First, let $h'{:}\, X' \to Y$ be the Galois closure of h with Galois group G. Let $\mathcal{D}$ be a semistable cover X' of such that $Y \notin \mathcal{C}$ where

$$\mathcal{C} = \{h'(U){:}\, U \in \mathcal{D}\}.$$

Then we claim $\mathcal{C}$ is a semi-stable cover of Y. Clearly $\mathcal{C}$ is a finite admissible open cover. By Lemma 1.7, if $W \in \mathcal{D}$ and $A \in E(W)$, $h'(A)$ is a wide open disk or annulus. Since $h'(W) \neq Y$, $h'(A)$ cannot be a disk for all $A \in E(W)$. It follows by a glueing argument, as in the proof of Lemma 1.2, that $h'(W)$ is a connected wide open. Now suppose $U, V \in \mathcal{D}$ and $h'(W) \neq h'(V)$. We must show $h'(W) \cap h'(V)$ is a finite union of disjoint annuli. First, we remark that $h'(W^u)$ and $h'(V^u)$ are disjoint affinoids in Y, using Lemma 1.6. Suppose A is a component of $W \cap V$ so $A \in E(W) \cap E(V)$. Suppose (x_n) is a sequence of points in A. If $x_n \to W^u$, $h'(x_n) \to h'(W^u)$ and if $x_n \to V^u$, $h'(x_n) \to h'(V^u)$. It follows that $h'(A)$ is an annulus. Also, we know that if U is a connected component of $h'(W) \cap h'(V)$, $U = \bigcup_{\sigma \in S} h'(A_\sigma)$ where the A_σ are in $E(W) \cap E(V^\sigma)$, for some subset S of G. Now it follows from Lemma 1.3 that if S has more than one element and $\sigma \in S$ there must be a $\tau \in S$ such that $\tau \neq \sigma$ and $A_\sigma \cap A_\tau \neq \emptyset$. Then $A_\sigma \cup A_\tau$ is an annulus, arguing as in the proof of Corollary 3.6a of [RLC] ($A_\sigma \cup A_\tau \neq Y$ since $W^u \cap (A_\sigma \cup A_\tau) = \emptyset$). The fact that A_σ and A_τ are connected to both W^u and V^u implies $A_\tau = A_\sigma \cup A_\tau = A_\sigma$. We conclude that all the A_σ equal U, for $\sigma \in S$ and so U is a wide open annulus.

Suppose $U, V, W \in \mathcal{D}$ are such that $h'(U), h'(V), h'(W)$ are distinct. If $y \in h'(U) \cap h'(V) \cap h'(W)$, there exist $\sigma, \tau \in G$ and $x \in U \cap V^\sigma \cap W^\tau$ such that $y = h'(x)$. But this implies U, V^σ, W^τ are not distinct which in turn implies $h'(U), h'(V), h'(W)$ are not distinct.

We must show for $U \in \mathcal{D}$, $h'(U)^u$, which equals

$$h'(U) \backslash (\bigcup_{\substack{V \in \mathcal{C} \\ V \neq h'(U)}} V),$$

is an affinoid whose reduction is irreducible and only has regular singular points. Now,

$$h'(U)^u = h'(\bigcup_{\sigma \in G} (U^\sigma \backslash \bigcup_{\substack{A \in E(U^\sigma) \cap E(V) \\ V \in \mathcal{D}, V \neq U^\tau, \tau \in G}} A)$$

and also

$$= h'(U^u) \cup \bigcup_{\substack{A \in E(U) \cap E(U^\sigma) \\ \sigma_{\neq 1} \in G,}} h'(A).$$

It follows from the first equality that $h'(U)^u$ is an affinoid using Lemma 1.6 and Proposition 3.3 of [RLC]. Its reduction is irreducible as the reduction U^u is, and from Proposition 1.5 it has at worst only regular singular points. Finally, since all the $h'(A)$ are disks or annuli by Lemma 1.7 whose ends are connected to $h'(U^u)$, $h(U)^u$ is an affinoid and these $h'(A)$ must, by Lemma 1.1, be smooth or regular singular classes of $h'(U)^u$, and thus, in particular, $h'(U)^u$ must have irreducible reduction. Thus $\mathcal{C}$ is a semi-stable cover and clearly h' induces a finite map of semi-stable covers from $\mathcal{D}$ to $\mathcal{C}$.

We also know X'/X is Galois and if $r\colon X' \to X$ is the corresponding morphism,

$$\mathcal{E} := \{r(U)\colon U \in \mathcal{D}\}$$

does not contain X so is a semi-stable cover of X and r induces a finite map of semi-stable covers from $\mathcal{D}$ to $\mathcal{E}$. It follows that h induces a finite map of semi-stable covers from $\mathcal{E}$ to $\mathcal{C}$ and hence extends to the corresponding semi-stable models.

Now we must explain how we can find a cover $\mathcal{D}$ of X' with the required properties. If X' has a stable model $\mathcal{X}$, then $\mathcal{X}$ is preserved by G. Let D be a wide open disk in X' such that $D^\sigma \cap D = \emptyset$ for all $\sigma_{\neq 1} \in G$ and B an affinoid ball in D. Let $\mathcal{X}'$ be the minimal semi-stable refinement of X such that no two elements of $\{D^\sigma\colon \sigma \in G\}$ are contained in the same residue class. Let $E = \bigcup_{\sigma \in G} B^\sigma$. Then we can take for $\mathcal{D}$,

$$\{(\mathrm{red}^{-1}\mathrm{Z})\backslash\mathrm{E}\colon \mathrm{Z} \text{ is an irreducible component of } \overline{\mathcal{X}}'\} \cup \{\mathrm{D}^\sigma\colon \sigma \in \mathrm{G}\}.$$

If $g(X') \leq 1$ and the set of ramified points $S \subset X'$ contains at least $3 - 2g(X')$ elements we do the same thing starting with the minimal semi-stable model with the property that S injects into the smooth points of the reduction of this model. The remaining cases are easier.

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